

INVESTIGATION OF THE STRESS STATE OF A MULTILAYER MEDIUM CONSISTING OF A LAYER, HALF-SPACE AND A CYLINDRICAL PIPE, WITH DISPLACEMENTS SPECIFIED ON THE BOUNDARY SURFACES

Miroshnikov V.Yu.

Keywords: composite, thick-walled pipe in half-space, coupling conditions, generalized Fourier method.

Abstract. A solution to the spatial problem of the theory of elasticity is proposed for a composite in the form of a half-space with a longitudinal thick-walled circular cylindrical tube and a layer rigidly attached to the surface of the half-space. Layer, half-space and pipe - elastic homogeneous isotropic materials different from each other.

On the free surface of the layer and the inner surface of the pipe, displacements are specified. At the boundary of the layer and half-space, as well as at the boundary of half-space and the outer surface of the pipe, the matching conditions are coupling. It is necessary to evaluate the stress state of the multilayer medium. The solution of the spatial problem of the theory of elasticity is obtained on the basis of the generalized Fourier method in cylindrical coordinates associated with the pipe and Cartesian coordinates associated with the layer and half-space.

A numerical analysis of the stress state of the composite body is carried out. Plots of normal and shear stresses in the layer, half-space and pipe are presented.

The proposed method can be used in the design of composites, structures and communications, provided that the statement of the problem coincides. Stress analysis can be used in the selection of geometric characteristics of the designed model.

ИССЛЕДОВАНИЕ НАПРЯЖЕННОГО СОСТОЯНИЯ МНОГОСЛОЙНОЙ СРЕДЫ, СОСТОЯЩЕЙ ИЗ СЛОЯ, ПОЛУПРОСТРАНСТВА И ЦИЛИНДРИЧЕСКОЙ ТРУБЫ, С ЗАДАНЫМИ НА ГРАНИЧНЫХ ПОВЕРХНОСТЯХ ПЕРЕМЕЩЕНИЯМИ

Мирошников В.Ю.

Ключевые слова: композит, толстостенная труба в полупространстве, условия сопряжения, обобщенный метод Фурье.

Аннотация. Предложено решение пространственной задачи теории упругости для композита в виде полупространства с продольной круглой цилиндрической толстостенной трубой и слоем, жестко прикрепленным к поверхности полупространства. Слой, полупространство и труба - упругие однородные изотропные материалы, отличающиеся друг от друга.

На свободной поверхности слоя и внутренней поверхности трубы заданы перемещения. На границе слоя и полупространства, а также на границе полупространства и внешней поверхности трубы условия сопряжения. Необходимо исследовать напряженное состояние многослойной среды. Решение пространственной задачи теории упругости получено на основе обобщенного метода Фурье в цилиндрических координатах, связанных с трубой, и декартовых координат, связанных со слоем и полупространством.

Проведен численный анализ напряженного состояния тела композита. Представлены графики нормальных и касательных напряжений в слое, полупространстве и трубе.

Предлагаемый метод может быть использован при проектировании композитов, конструкций и коммуникаций при условии совпадения постановки задачи. Анализ напряженного состояния может быть использован при выборе геометрических характеристик проектируемой модели.

Introduction

When designing complex geotechnical structures, as well as structures and parts made of composites, one has to deal with design schemes in the form of rigidly interlinked layers, pipes and half-space. In view of the combination of different boundary surfaces and the spatial formulation of the problem, the solution is much more complicated.

For such problems, the generalized Fourier method [1] is used in the numerical analytical approach.

So, using this method, the problems are solved for a half-space, with a surface parallel to it, a cylindrical cavity or inclusion [2–8], for a cylinder with cylindrical inclusions [9], for a layer with one cylindrical cavity, inclusion or pipe [10–12].

The presented work is also based on the generalized Fourier method.

Formulation of the problem

In an elastic homogeneous half-space parallel to its surface, there is a circular cylindrical thick-walled pipe with an outer radius of R_1 and an inner one of R_2 . A layer of height h_1 is rigidly adhered to the surface of the half-space.

We will consider the pipe in a cylindrical coordinate system (ρ, φ, z) , the half-space in the Cartesian coordinate system (x_2, y_2, z_2) , which is identically oriented and combined with the coordinate system of the pipe. The half-space boundary is located at $y_2=h_2$. The layer will be considered in the Cartesian coordinate system (x_1, y_1, z_1) located on the lower surface of the layer (the boundary of the coupling with half-space and, accordingly, shifted relative to the coordinate system by $y_2=h_2$).

It is necessary to find a solution to the Lamé equations $\Delta \vec{U}_j + (1 - 2\sigma_j)^{-1} \nabla \text{div} \vec{U}_j = 0$, where σ_j – Poisson's ratio of the layer ($j=1$), half-space ($j=2$) or pipes ($j=3$).

At the upper boundary of the layer, displacements are specified $\vec{U}_1(x_1, z_1)|_{y_1=h_1} = \vec{U}_h^0(x_1, z_1)$, displacements are set on the inner surface of the pipe $\vec{U}_2(\varphi, z)|_{\rho=R_2} = \vec{U}_R^0(\varphi, z)$, where $\vec{U}_1$ – displacements in layer; $\vec{U}_2$ – displacements in half-space;

$$\begin{aligned} \vec{U}_h^0(x_1, z_1) &= U_x^{(1)} \vec{e}_1^{(1)} + U_y^{(1)} \vec{e}_2^{(1)} + U_z^{(1)} \vec{e}_3^{(1)}, \\ \vec{U}_R^0(\varphi, z) &= U_\rho^{(R)} \vec{e}_1^{(2)} + U_\varphi^{(R)} \vec{e}_2^{(2)} + U_z^{(R)} \vec{e}_3^{(2)} \end{aligned} \quad (1)$$

known functions; $\vec{e}_j^{(k)}$, $j = 1, 2, 3$ – the unit vectors of the Cartesian ($k=1$) and cylindrical ($k=2$) coordinate systems.

On the boundary of the layer and half-space, the coupling conditions are given

$$\vec{U}_1|_{y_1=0} = \vec{U}_2|_{y_2=h_2}, \quad (2)$$

$$F_1 \vec{U}_1|_{y_1=0} = F_2 \vec{U}_2|_{y_2=h_2}, \quad (3)$$

at the boundary of the half-space and the pipe, the coupling conditions

$$\vec{U}_2(\varphi, z)|_{\rho=R_1} = \vec{U}_3(\varphi, z)|_{\rho=R_1}, \quad (4)$$

$$F\vec{U}_2(\varphi, z)|_{\rho=R_1} = F\vec{U}_3(\varphi, z)|_{\rho=R_1}, \quad (5)$$

where $F_j\vec{U}_j| = 2G_j[\frac{\sigma_j}{1-2\sigma_j}\vec{n}\operatorname{div}U_j + \frac{\partial}{\partial n}\vec{U}_j + \frac{1}{2}(\vec{n}\times\operatorname{rot}\vec{U}_j)]$; $G_j = \frac{E_j}{2(1+\sigma_j)}$; σ_j , E_j – Poisson's ratio and modulus of elasticity of the layer ($j=1$), half-space ($j=2$) or pipes ($j=3$); $\vec{U}_3$ – displacements in pipes.

The specified functions will be considered as rapidly decreasing from the origin of coordinates along the z axis for the cylinder and along the z and x axes for the layer boundaries.

Solution Method

We take the basic solutions of the Lamé equation in the form [1]

$$\begin{aligned} \vec{u}_k^{\pm}(x, y, z; \lambda, \mu) &= N_k^{(d)} e^{i(\lambda z + \mu x) \pm \gamma y}; \\ \vec{R}_{k,m}(\rho, \varphi, z; \lambda) &= N_k^{(p)} I_m(\lambda \rho) e^{i(\lambda z + m\varphi)}; \\ \vec{S}_{k,m}(\rho, \varphi, z; \lambda) &= N_k^{(p)} [(\operatorname{sign} \lambda)^m K_m(|\lambda| \rho) \cdot e^{i(\lambda z + m\varphi)}]; \quad k=1, 2, 3; \\ N_1^{(d)} &= \frac{1}{\lambda} \nabla; \quad N_2^{(d)} = \frac{4}{\lambda} (\sigma - 1) \vec{e}_2^{(1)} + \frac{1}{\lambda} \nabla(y \cdot); \quad N_3^{(d)} = \frac{i}{\lambda} \operatorname{rot}(\vec{e}_3^{(1)} \cdot); \quad N_1^{(p)} = \frac{1}{\lambda} \nabla; \\ N_2^{(p)} &= \frac{1}{\lambda} \left[\nabla \left(\rho \frac{\partial}{\partial \rho} \right) + 4(\sigma - 1) \left(\nabla - \vec{e}_3^{(2)} \frac{\partial}{\partial z} \right) \right]; \quad N_3^{(p)} = \frac{i}{\lambda} \operatorname{rot}(\vec{e}_3^{(2)} \cdot); \\ \gamma &= \sqrt{\lambda^2 + \mu^2}, \quad -\infty < \lambda, \mu < \infty, \end{aligned} \quad (6)$$

where $I_m(x)$, $K_m(x)$ – modified Bessel functions; $\vec{R}_{k,m}$, $\vec{S}_{k,m}$, $k=1, 2, 3$ – respectively, internal and external solutions of the Lamé equation for the cylinder; $\vec{u}_k^{(-)}$, $\vec{u}_k^{(+)}$ – solutions of the Lamé equation for a layer and half-space.

The solution to the problem is presented in the form

$$\begin{aligned} \vec{U}_1 &= \sum_{k=1}^3 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (H_k^{(1)}(\lambda, \mu) \cdot \vec{u}_k^{(+)}(x_1, y_1, z_1; \lambda, \mu; \sigma_1) + \\ &+ \tilde{H}_k^{(1)}(\lambda, \mu) \cdot \vec{u}_k^{(-)}(x_1, y_1, z; \lambda, \mu; \sigma_1)) d\mu d\lambda, \end{aligned} \quad (7)$$

$$\begin{aligned} \vec{U}_2 &= \sum_{k=1}^3 \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} B_{k,m}(\lambda) \cdot \vec{S}_{k,m}(\rho, \varphi, z; \lambda; \sigma_1) d\lambda + \\ &+ \sum_{k=1}^3 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (H_k^{(2)}(\lambda, \mu) \cdot \vec{u}_k^{(+)}(x_2, y_2, z_2; \lambda, \mu; \sigma_2)) d\mu d\lambda, \end{aligned} \quad (8)$$

$$\vec{U}_3 = \sum_{k=1}^3 \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} A_{k,m}(\lambda) \cdot \vec{R}_{k,m}(\rho, \varphi, z; \lambda) + \tilde{A}_{k,m}(\lambda) \cdot \vec{S}_{k,m}(\rho, \varphi, z; \lambda) d\lambda, \quad (9)$$

where $\vec{S}_{k,m}(\rho, \varphi, z; \lambda)$, $\vec{R}_{k,m}(\rho, \varphi, z; \lambda)$, $\vec{u}_k^{(+)}(x, y, z; \lambda, \mu)$ и $\vec{u}_k^{(-)}(x, y, z; \lambda, \mu)$ – basic solutions given by formulas (6), unknown functions $H_k^{(1)}(\lambda, \mu)$, $\tilde{H}_k^{(1)}(\lambda, \mu)$, $B_{k,m}(\lambda)$, $H_k^{(2)}(\lambda, \mu)$, $A_{k,m}(\lambda)$ и $\tilde{A}_{k,m}(\lambda)$ must be found from the boundary conditions (1) and coupling conditions (2 – 5).

For the transition of basic solutions between coordinate systems, we use the formulas [12].

To satisfy the boundary conditions at the upper boundary of the layer, we equate vector (7), for $y_1=h_1$, to the given one $\vec{U}_h^0(x, z)$, represented through the double Fourier integral. So we get three equations (one for each projection) with six unknowns $H_k^{(1)}(\lambda, \mu)$, $\tilde{H}_k^{(1)}(\lambda, \mu)$.

To satisfy the coupling conditions at the boundary of the layer and half-space in displacements, we substitute the right-hand sides (7) and (8) in (2). In this case, writing expression $\vec{U}_2(x_2, z_2)_{|y_2=0}$, it is necessary to use the formulas for the transition from solutions $\vec{S}_{k,m}$ of the cylinder to solutions $\vec{u}_k^{(-)}$ [12, formula (7)]. In a similar way, we can write three additional equations for stresses (3).

So we get nine infinite systems of equations with unknown functions $H_k^{(1)}(\lambda, \mu)$, $\tilde{H}_k^{(1)}(\lambda, \mu)$, $H_k^{(2)}(\lambda, \mu)$ и $B_{k,m}(\lambda)$.

This system of equations has a unique solution, since the determinant Δ of this system has the form $\Delta = -16 \cdot \gamma^6 \cdot e^{-3\gamma(h_1-h_2)} \cdot \Phi(\gamma) / \lambda^6$, where $\Phi(\gamma)$ – the function, for $\gamma > 0$, has only positive values and does not vanish.

From the resulting system of equations, we express $H_k^{(1)}(\lambda, \mu)$, $\tilde{H}_k^{(1)}(\lambda, \mu)$ and $H_k^{(2)}(\lambda, \mu)$ through $B_{k,m}(\lambda)$.

To satisfy the coupling conditions at the boundary of the half-space and the pipe, we set $\rho=R_1$ in (8) and (9). We expand in (8) the basic solutions of $\vec{u}_k^{(+)}$ by the solutions of $\vec{R}_{k,m}$ using the transition formulas [12, formula (8)]. The resulting vector, as well as vector (9), for $\rho=R_1$, we substitute in (4). So we get three infinite systems of equations for the coupling of half-space and pipe in displacements. Similarly, we write three equations for stresses (5).

To fulfill the boundary conditions on the inner surface of the pipe, we equate (9), for $\rho=R_2$, given by $\vec{U}_R^0(\varphi, z)$, represented by the integral and the Fourier series.

Having received 9 infinite equations, instead of $H_k^{(2)}(\lambda, \mu)$, we substitute the previously expressed functions through $B_{k,m}(\lambda)$, free ourselves from the series in m and the integrals in λ . As a result, we obtain a set of nine infinite systems of linear algebraic equations for determining unknown $B_{k,m}(\lambda)$, $A_{k,m}(\lambda)$ and $\tilde{A}_{k,m}(\lambda)$. These infinite systems have the properties of equations of the second kind and, as a consequence, the reduction method can be applied to them.

Having solved this system of equations, we find the unknowns $A_{k,m}(\lambda)$, $\tilde{A}_{k,m}(\lambda)$ and $B_{k,m}(\lambda)$.

Found from an infinite system of equations $B_{k,m}(\lambda)$, we substitute in the expressions for $H_k^{(1)}(\lambda, \mu)$, $\tilde{H}_k^{(1)}(\lambda, \mu)$ and $H_k^{(2)}(\lambda, \mu)$. This will identify all unknown tasks.

Numerical Studies of Stress State

There is an elastic isotropic half-space in which, parallel to its surface, there is a round cylindrical thick-walled pipe. A layer is rigidly adhered to the surface of the half-space. The layer material is plastic, Poisson's ratio $\sigma_1 = 0.38$, elastic modulus $E_1 = 1700 \text{ N/mm}^2$. Half-space - concrete, Poisson's ratio $\sigma_2 = 0.16$, elastic modulus $E_2 = 32500 \text{ N/mm}^2$. The pipe is steel, Poisson's ratio $\sigma_3 = 0.25$, elastic modulus $E_3 = 200000 \text{ N/mm}^2$. The outer radius of the pipe $R_1 = 30 \text{ mm}$, the inner $R_2 = 20 \text{ mm}$. The distance from the upper boundary of the layer to the center of the pipe is $h_2 = 45 \text{ mm}$. Layer thickness $h_1 = 10 \text{ mm}$.

On the inner surface of the pipe, displacements in the form $U_\rho^{(R)} = 10^4 \cdot (z^2 + 10^2)^{-2}$, $U_\phi^{(R)} = U_z^{(R)} = 0$. At the upper boundary of the layer, displacements are specified $U_y^{(1)} = U_x^{(1)} = U_z^{(1)} = 0$.

The infinite system was truncated to $m = 8$. The integrals are calculated using Philon's quadrature formulas (for oscillating functions) and Simpson's ones (for functions without oscillations). The accuracy of the boundary conditions, for the specified values of m and given geometric parameters, is 10^{-4} .

Figure 1 shows the stress state on the inner surface of the pipe (Fig. 1a) and in the body of the half-space on the interface with the pipe (Fig. 1b), at $z = 0$ in N/mm^2 .

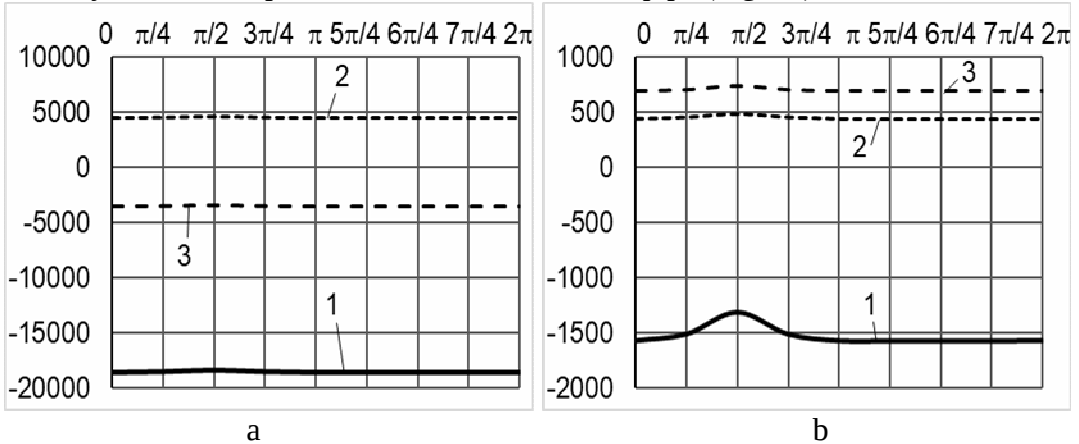


Fig. 1. Stress state: a – on the surface $\rho=R_2$; b – on the surface $\rho=R_1$;

1 – σ_ρ , 2 – σ_ϕ , 3 – σ_z

On the inner surface of the pipe (Fig. 1a), the stresses are almost constant, but on the interface of the pipe and half-space (Fig. 1b), the stresses not only

substantially decrease in general, but they also have a clear effect of reducing stresses in the zone close to the half-space boundary ($\varphi=\pi/2$).

Figure 2 shows the stresses along the y axis on the isthmus between the upper point of the cavity and the interface between the half-space and the layer, at $z = 0$, in N/mm^2 .

In the body of a half-space, maximum stresses σ_p are observed on the surface of the cavity (Fig. 2, line 1) and decrease with approaching the boundary of the half-space. The maximum stresses σ_φ (Fig. 2, line 2) and σ_z (Fig. 2, line 3) are also located on the surface of the cavity, they decrease as they approach the boundary of the half-space, and near the coupling between the layer and the layer they increase.

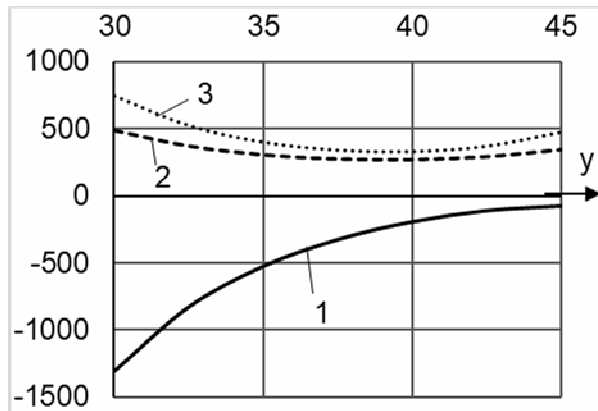


Fig. 2. Stresses along the y axis between the surface of the cavity and the boundary of the half-space: 1 – σ_p , 2 – σ_φ , 3 – σ_z

Figure 3 shows the stresses at the upper boundary of the layer (Fig. 3a) and at the coupling between the layer and the half-space (in the layer body, Fig. 3b), along the z axis, at $x = 0$, in N/mm^2 .

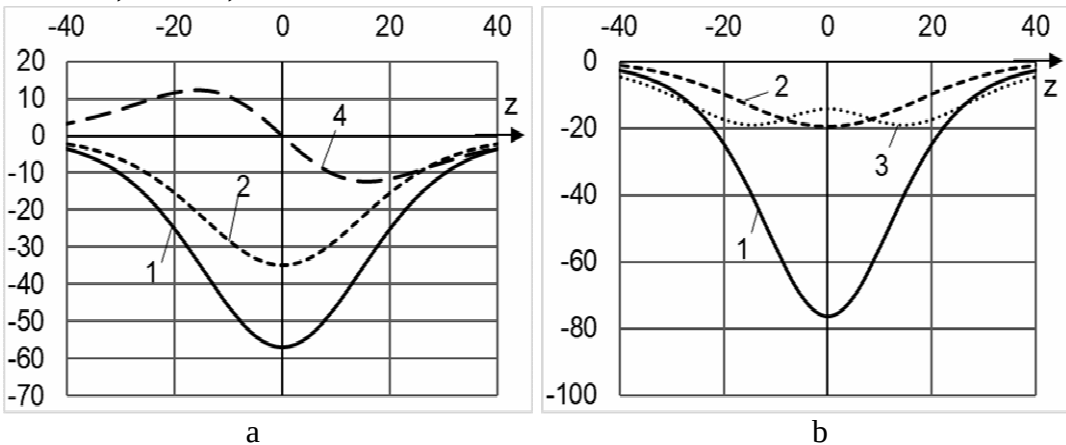


Fig. 3. Stresses in the layer: a – on the upper boundary of the layer; b – at the bottom of the layer; 1 – σ_y , 2 – σ_x , 3 – σ_z , 4 – τ_{yz}

On the upper surface of the layer, the stresses σ_x and σ_z have very close values, as a result of which the voltage σ_z is omitted on the graph (Fig. 3a).

The greatest stresses in the layer, both in the upper and lower parts, are σ_y (Fig. 3, line 1).

Due to hard pinching of the upper part of the layer, the stresses σ_x and σ_z in the layer have negative values (lines 2, 3).

Summary

Based on the generalized Fourier method, a method is proposed for solving the spatial problem of the theory of elasticity for a multilayer medium in the form of a layer and a half-space in which a longitudinal cylindrical pipe is located. The problem is reduced to an infinite system of linear algebraic equations, allowing the application of the reduction method to it. Numerical studies give reason to assert that its solution can be found with any accuracy by the proposed method, which is confirmed by the high accuracy of fulfilling the boundary conditions.

The solution method can be used in the design of composites, machine parts, building structures and underground structures, the design scheme of which coincides with the formulation of this problem.

The presented stress state analysis can be used to select the geometric parameters of the structure.

The presented stress state analysis can be used to select the geometric parameters of the structure.

Список литературы

1. Николаев А.Г., Проценко В.С. Обобщенный метод Фурье в пространственных задачах теории упругости. Харьков: Нац. аэрокосм. университет им. Н.Е. Жуковского «ХАИ», 2011. 344 с.
2. Николаев А.Г., Орлов Е.М. Решение первой осесимметричной термоупругой краевой задачи для трансверсально-изотропного полупространства со сфероидальной полостью // Проблемы вычислительной механики и прочности конструкций. 2012. Вып. 20. С. 253-259.
3. Miroshnikov V.Yu. First basic elasticity theory problem in a half-space with several parallel round cylindrical cavities // Journal of Mechanical Engineering. 2018. Vol.21, №. 2. P. 12-18.
4. Protsenko V., Miroshnikov V. Investigating a problem from the theory of elasticity for a half-space with cylindrical cavities for which boundary conditions of contact type are assigned // Eastern-European Journal of Enterprise Technologies. Applied mechanics. 2018. Vol. 4, №7, P. 43-50. doi: <https://doi.org/10.15587/1729-4061.2018.139567>
5. Николаев А. Г., Щербак А. Ю., Юхно А. И. Действие сосредоточенной силы на трансверсально-изотропное полупространство с параболоидальным включением // Вопросы проектирования и

производства конструкций летательных аппаратов: Сб. науч. тр. Нац. аэрокосм. ун-та им. Н. Е. Жуковского «ХАИ». 2006. Вып. 2. С. 47-51.

6. Miroshnikov V.Yu. Evaluation of the stress-strain state of half-space with cylindrical cavities // *Visnyk Dniprovs'koho universytetu. Seriya: Mekhanika – Bulletin of the Dnipro University. Series: Mechanics*. 2018. Vol. 26, № 5. P. 109-118.
7. Проценко В.С., Українець Н.А. Применение обобщенного метода Фурье к решению первой основной задачи теории упругости в полупространстве с цилиндрической полостью // *Вестник Запорожского национального университета*. 2015. Вып. 2. С. 193-202.
8. Проценко В.С., Попова Н.А. Вторая основная краевая задача теории упругости для полупространства с круговой цилиндрической полостью // *Доклады НАН Украины*. – 2004. – № 12. – С. 52-58.
9. Николаев А.Г., Танчик Е.А. Распределение напряжений в ячейке однонаправленного композиционного материала, образованного четырьмя цилиндрическими волокнами // *Вестник Одесского национального университета. Математика. Механика*. 2013. Т.18, Вып. 4(20). С. 101-111.
10. Мирошников В.Ю. Исследование второй основной задачи теории упругости для слоя с цилиндрической полостью // *Сопротивление материалов и теория сооружений*. 2019. №102. С. 77–90.
11. Мирошников В.Ю. Смешанная задача теории упругости для слоя с цилиндрическими включениями // *Научный вестник строительства*. 2019. Т. 96, №2. С. 247–252. DOI: 10.29295/2311–7257–2019–96–2–247–252
12. Miroshnikov V. Investigation of the Stress Strain State of the Layer with a Longitudinal Cylindrical Thick-Walled Tube and the Displacements Given at the Boundaries of the Layer // *Journal of Mechanical Engineering*. 2019. Vol. 22, N 2. P. 44-52. <https://doi.org/10.15407/pmach2019.02.044>

References

1. Nikolayev A.G., Protsenko V.S. The generalized Fourier method in spatial problems of the theory of elasticity. Kharkov: Nats. aerokosm. universitet im. N.Ye. Zhukovskogo «KHAI», 2011. 344 p.
2. Nikolayev A.G., Orlov Ye.M. Solution of the first axisymmetric thermoelastic boundary value problem for a transversely isotropic half-space with a spheroidal cavity // *Problems of computational mechanics and structural durability*. 2012, vol. 20, pp. 253-259.
3. Miroshnikov V.Yu. First basic elasticity theory problem in a half-space with several parallel round cylindrical cavities // *Journal of Mechanical Engineering*. 2018. Vol.21, №. 2. P. 12-18.
4. Protsenko V., Miroshnikov V. Investigating a problem from the theory of elasticity for a half-space with cylindrical cavities for which boundary conditions of contact type are assigned // *Eastern-European Journal of Enterprise Technologies. Applied mechanics*. 2018. Vol. 4, №7, P. 43-50. doi: <https://doi.org/10.15587/1729-4061.2018.139567>

5. Nikolayev A.G., Shcherbakova A.Yu., Yukhno A.I. Action of Concentrated Force on a Transversely-Isotropic Half-Space with Paraboloidal Inclusion // Issues of Design and Production of Structures of Aircraft. 2006, vol. 2, pp. 47-51.
6. Miroshnikov V.Yu. Evaluation of the stress-strain state of half-space with cylindrical cavities // Bulletin of the Dnipro University. Series: Mechanics. 2018, vol. 26, no. 5, pp. 109 – 118.
7. Protsenko V.S., Ukrainets N.A. Application of the generalized Fourier method to the solution of the first main problem of the theory of elasticity in a half-space with a cylindrical cavity // Bulletin of the Zaporizhzhya National University. 2015, vol. 2, pp. 193-202.
8. Protsenko V.S., Ukrainets N.A. The second main boundary value problem of the theory of elasticity for a half-space with a circular cylindrical cavity // Reports of the National Academy of Sciences of Ukraine. 2004, vol. 12, pp. 52-58.
9. Nikolayev A.G., Tanchik Ye.A. Stress distribution in a cell of a unidirectional composite material formed by four cylindrical fibers // Bulletin of the Odessa National University. Maths. Mechanics. 2013, vol. 4, pp. 101-111.
10. Miroshnikov V.Yu. Investigation of the second main problem of the theory of elasticity for a layer with a cylindrical cavity // Materials resistance and theory of structures. 2019. №102. P. 77-90.
11. Miroshnikov V.Yu. A mixed problem of elasticity theory for a cylindrical inclusion layer // Scientific Bulletin of Construction. 2019, Vol 2, №2 (96). P. 247-252. DOI: 10.29295/2311–7257–2019–96–2–247–252
12. Miroshnikov V. Investigation of the Stress Strain State of the Layer with a Longitudinal Cylindrical Thick-Walled Tube and the Displacements Given at the Boundaries of the Layer // Journal of Mechanical Engineering. 2019. Vol. 22, N 2. P. 44-52. <https://doi.org/10.15407/pmach2019.02.044>

Мирошников Виталий Юрьевич – кандидат технических наук, доцент, Харьковский национальный университет строительства и архитектуры, г. Харьков, Украина, m0672628781@gmail.com	Miroshnikov Vitaly Yuryevich – candidate of technical sciences, associate professor, Kharkov National University of Construction and Architecture, Kharkov, Ukraine, m0672628781@gmail.com
---	---

Received 14.09.2019